Non-linear condition that is consistent with the general theory of relativity

23th/Sep/2016

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1. Overview

I Introduced the spherical wave solution using a non-linear function to that is depending on the electric potential (scalar potential) in March 2016. However, consistent with the Lorentz invariance and general relativity was not able to explain.

So I was focused on the relationship of the Ricci tensor and the electromagnetic energy of the general relativity theory.

I considered that the non-linear standing wave solution of electromagnetic field is present in the strong electromagnetic field.

I tried looking for a numerical solution.

2. Non-linear standing wave solution

We think of a solution, such as the following.

ϕ : Scalar potential

r : Radius

ω : Frequency corresponding to the Compton wavelength

c : speed of light

t : time

ϕ_0 :Constant the representative of the amplitude of scalar potential

ϕ_n :A constant representing the magnitude of the random noise

E Electric field intensity of radial direction

And there is a standing wave scalar potential to form particles around the origin.

In the vicinity of the center, It is non-linear because of the strong electromagnetic field.

In contrast, it is almost linear in the peripheral region there is a spherical standing wave as shown below.

$$\phi = \frac{\phi_0}{r}\cos\left(\frac{\omega r}{c}\right)\exp(i\omega t) + \phi_n \qquad (1)$$

However ϕ_n is the random noise

Therefore

$$\phi = \frac{\phi_0}{r}\cos\left(\frac{\omega r}{c}\right)\exp(i\omega t) + \phi_n = \frac{\phi_0}{r}\left(\cos\left(\frac{\omega r}{c}\right)\right)(\cos(\omega t) + i\sin(\omega t)) + \phi_n$$

$$= \frac{\phi_0}{r}\left(\cos\left(\frac{\omega r}{c}\right)\cos(\omega t)\right) + i\frac{\phi_0}{r}\left(\cos\left(\frac{\omega r}{c}\right)\sin(\omega t)\right) + \phi_n \qquad (2)$$

Since the electric field strength can be obtained by this gradient

$$E = \frac{\partial \phi}{\partial r} = -\frac{\phi_0}{r^2}\cos\left(\frac{\omega r}{c}\right)\exp\left(i\omega t\right) + \left(\frac{\omega}{c}\right)\frac{\phi_0}{r}\cos\left(\frac{\omega r}{c}\right)\exp\left(i\omega t\right) + \nabla \phi_n \quad (3)$$

$$\left|E\right|^2 = \frac{{\phi_0}^2}{2r^4} + \left(\frac{\omega^2}{c^2}\right)\frac{{\phi_0}^2}{2r^2} + \left|\nabla \phi_n\right|^2 \quad (4)$$

In the entire region including the non-linear region and the linear region

ϕ_p is amplitude of the scalar potential

$$\phi = \phi_p \exp\left(-i\omega t\right) + \phi_n \quad (5)$$

$$E = \frac{\partial \phi_p}{\partial r}\exp\left(-i\omega t\right) + \nabla \phi_n \quad (6)$$

$$\left|E\right|^2 = \left|\frac{\partial \phi_p}{\partial r}\right|^2 + \left|\nabla \phi_n\right|^2 \quad (7)$$

In the linear region

$$\phi_p = \frac{\phi_0}{r}\cos\left(\frac{\omega r}{c}\right) \quad (8)$$

3. Non-linear condition that from the general theory of relativity

Here we examine the relationship between the Ricci tensor and the electromagnetic energy of the general relativity theory.

From 'general relativity theory` by Tatsuo Uchiyama,

A^{λ} :Electromagnetic four-potential
j^{λ} :Four-current density
$E^{\mu\upsilon}$: Energy tensor density of the electromagnetic field
$T^{\mu\upsilon}$:Energy tensor density
$R^{\mu\nu}$:Ricci tensor is
G : Universal gravitation constant

$$\Box A^{\lambda} + R^{\lambda}_{\rho} A^{\rho} = -\mu_0 j^{\lambda} \qquad \text{(19.25) p115}$$

$$E^{0}_{0} = \frac{1}{2}\varepsilon_0 |E|^2 + \frac{1}{2\mu_0}|B|^2 \qquad \text{p114}$$

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = \kappa T^{\mu\nu} \qquad \text{(21.8)p123}$$

$$-R = \kappa T \qquad \text{(21.10) p124}$$

$$\kappa = \frac{8\pi}{c^4} G \qquad \text{(21.9) p123}$$

$$T^{00} = -\rho c^2 \quad \text{p126}$$

$$T \cong \rho c^2 \quad \text{p126}$$

With assuming weak gravitational field, we approximating $g^{00} \cong -1$

$$T^{00} = E^{00} = -E^{0}_{0}$$

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = R^{\mu\nu} - \frac{1}{2}\kappa T = \kappa T^{\mu\nu}$$

$$R^{00} - \frac{1}{2}\kappa T = R^{00} + \frac{1}{2}\kappa T^{00} = -\kappa T^{00}$$

$$R^{00} = -\frac{1}{2}\kappa T^{00} = -\frac{1}{2}\frac{8\pi}{c^4} G E^{00} = \frac{1}{2}\frac{8\pi}{c^4} G E^{0}_{0} = \frac{1}{2}\frac{8\pi}{c^4} G \left(\frac{1}{2}\varepsilon_0 |E|^2 + \frac{1}{2\mu_0}|B|^2 \right)$$

$$= \frac{4\pi}{c^4} G \left(\frac{1}{2} \varepsilon_0 |E|^2 + \frac{1}{2\mu_0} |B|^2 \right)$$

$$R^0_{\ 0} = g_{0\upsilon} R^{0\upsilon} = -R^{00}$$

We assume the electromagnetic field is the longitudinal wave

And we assumed as

$B = 0$

$j^{\lambda} = 0$

Then

$$\Box A^0 = -R^{\lambda}_{\ \rho} A^{\rho} = R^{00} A^0 = \frac{4\pi}{c^4} G \left(\frac{1}{2} \varepsilon_0 |E|^2 \right) A^0$$

$$\phi = A^0 \text{ とし}$$

$$\frac{\Box \phi}{\phi} = \frac{2\pi}{c^4} G \left(\varepsilon_0 |E|^2 \right) \qquad (9)$$

d'Alembert operator of the scalar potential is proportional to the square of the electric field strength that is indicating non-linearity.

However d'Alembert operator is $\Box \equiv \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}$

We assume that $\frac{\Box \phi}{\phi} = -K |E|^2$

then

$$K = \frac{2\pi G \varepsilon_0}{c^4} \qquad (10)$$

4. Numerical solution of standing wave solution

From(5) (9)

$$\Box\phi = \Box\left(\phi_p \exp(-i\omega t)+\phi_n\right) = \left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\left(\phi_p \exp(-i\omega t)+\phi_n\right)$$

$$= \nabla^2\left(\phi_p \exp(-i\omega t)+\phi_n\right) + \frac{\omega^2}{c^2}\left(\phi_p \exp(-i\omega t)+\phi_n\right) \tag{11}$$

$$\Box\phi = -K|E|^2\left(\phi_p \exp(-i\omega t)+\phi_n\right) = -K\left(\left|\frac{\partial\phi_p}{\partial r}\right|^2 + |\nabla\phi_n|^2\right)\left(\phi_p \exp(-i\omega t)+\phi_n\right) \tag{12}$$

With using Complex Fourier transform

From(11)

$$\frac{\omega}{2\pi}\int_{t'=t}^{t+\frac{2\pi}{\omega}} \Box\phi \exp(i\omega t')dt' = \frac{\omega}{2\pi}\int_{t'=t}^{t+\frac{2\pi}{\omega}} \Box\left(\phi_p \exp(-i\omega t')+\phi_n\right)\exp(i\omega t')dt' = \frac{\omega}{2\pi}\int_{t'=t}^{t+\frac{2\pi}{\omega}}\left(\nabla^2\phi_p + \frac{\omega^2}{c^2}\phi_p\right)dt'$$

$$= \nabla^2\phi_p + \frac{\omega^2}{c^2}\phi_p$$

From(12)

$$\frac{\omega}{2\pi}\int_{t'=t}^{t+\frac{2\pi}{\omega}} \Box\phi \exp(i\omega t')dt' = \frac{\omega}{2\pi}\int_{t'=t}^{t+\frac{2\pi}{\omega}} -K|E|^2\left(\phi_p \exp(-i\omega t)+\phi_n\right)\exp(i\omega t')dt'$$

$$= \frac{\omega}{2\pi}\int_{t'=t}^{t+\frac{2\pi}{\omega}} -K\left(\left|\frac{\partial\phi_p}{\partial r}\right|^2 + |\nabla\phi_n|^2\right)\left(\phi_p \exp(-i\omega t)+\phi_n\right)\exp(i\omega t')dt'$$

$$= -K\left(\left|\frac{\partial\phi_p}{\partial r}\right|^2 + |\nabla\phi_n|^2\right)\phi_p$$

Therefore

$$\nabla^2\phi_p + \frac{\omega^2}{c^2}\phi_p = -K\left(\left|\frac{\partial\phi_p}{\partial r}\right|^2 + |\nabla\phi_n|^2\right)\phi_p \tag{13}$$

Since the Laplacian on polar coordinates is $\nabla^2\phi_p = \frac{\partial^2\phi_p}{\partial r^2} + \frac{2}{r}\frac{\partial\phi_p}{\partial r}$

$$\frac{\partial^2\phi_p}{\partial r^2} = -\frac{2}{r}\frac{\partial\phi_p}{\partial r} - \frac{\omega^2}{c^2}\phi_p - K\left(\left|\frac{\partial\phi_p}{\partial r}\right|^2 + |\nabla\phi_n|^2\right)\phi_p \tag{14}$$

Here we will determine the radius r as a approximate boundary of linear and non-linear.

From(14) we assume $\left|K\left|\frac{\partial\phi_p}{\partial r}\right|^2\phi_p\right| = \left|\frac{\omega^2}{c^2}\phi_p\right|$

$$K\left|\frac{\partial\phi_p}{\partial r}\right|^2 = \frac{\omega^2}{c^2}$$

We use the formula of linear

$$\text{When } \frac{\omega}{c} << r$$

$$\left|\frac{\partial\phi_p}{\partial r}\right| = \left|\frac{\partial\left(\frac{\phi_0}{r}\exp\left(i\frac{\omega r}{c}\right)\right)}{\partial r}\right| \cong \frac{\omega}{c}\frac{\phi_0}{r}$$

therefore

$$K\frac{\omega^2}{c^2}\frac{\phi_0}{r^2}^2 = \frac{\omega^2}{c^2}$$

And the radius of this time is r_0

$$r_0^2 = K\phi_0^2$$

$$r_0 = \phi_0\sqrt{K} \qquad (15)$$

Finally an approximate boundary of the linear and non-linear is obtained as r_0.

This boundary r_0 is assumed to be $\frac{1}{k_r}$ times the Compton wavelength

However k_r is a constant

$$r_0 = \frac{2\pi c}{\omega k_r} \qquad (16)$$

$$\phi_0 = \frac{r_0}{\sqrt{K}} \qquad (17)$$

The amplitude scalar potential ϕ_0 is obtained by determining the k_r.
So we can get a dimensionless adjustment parameter k_r.
We assignment (14) to the difference equation as follows

$$\frac{\partial\phi_p(r-\Delta r)}{\partial r} = \frac{\partial\phi_p(r)}{\partial r} - \frac{\partial^2\phi_p(r)}{\partial r^2}\Delta r \qquad (18)$$

$$\phi_p(r-\Delta r)=\phi_p(r)-\frac{\partial\phi_p(r)}{\partial r}\Delta r \qquad (19)$$

The curve of ϕ_p using the spreadsheet software by this formula was drawn on the graph.

We adjust parameters by trial and error.

When we adjust $k_r = 1.75$ and $|\nabla \phi_n|^2 = 3.5 \times 10^{75}$

a solution was found to converge by the radius = 0.

In the Figure 1 the horizontal axis is radius, the upper part is scalar potential of case of non-linear and linear.

The lower part is the square of the electric field strength that indicates nonlinearity.

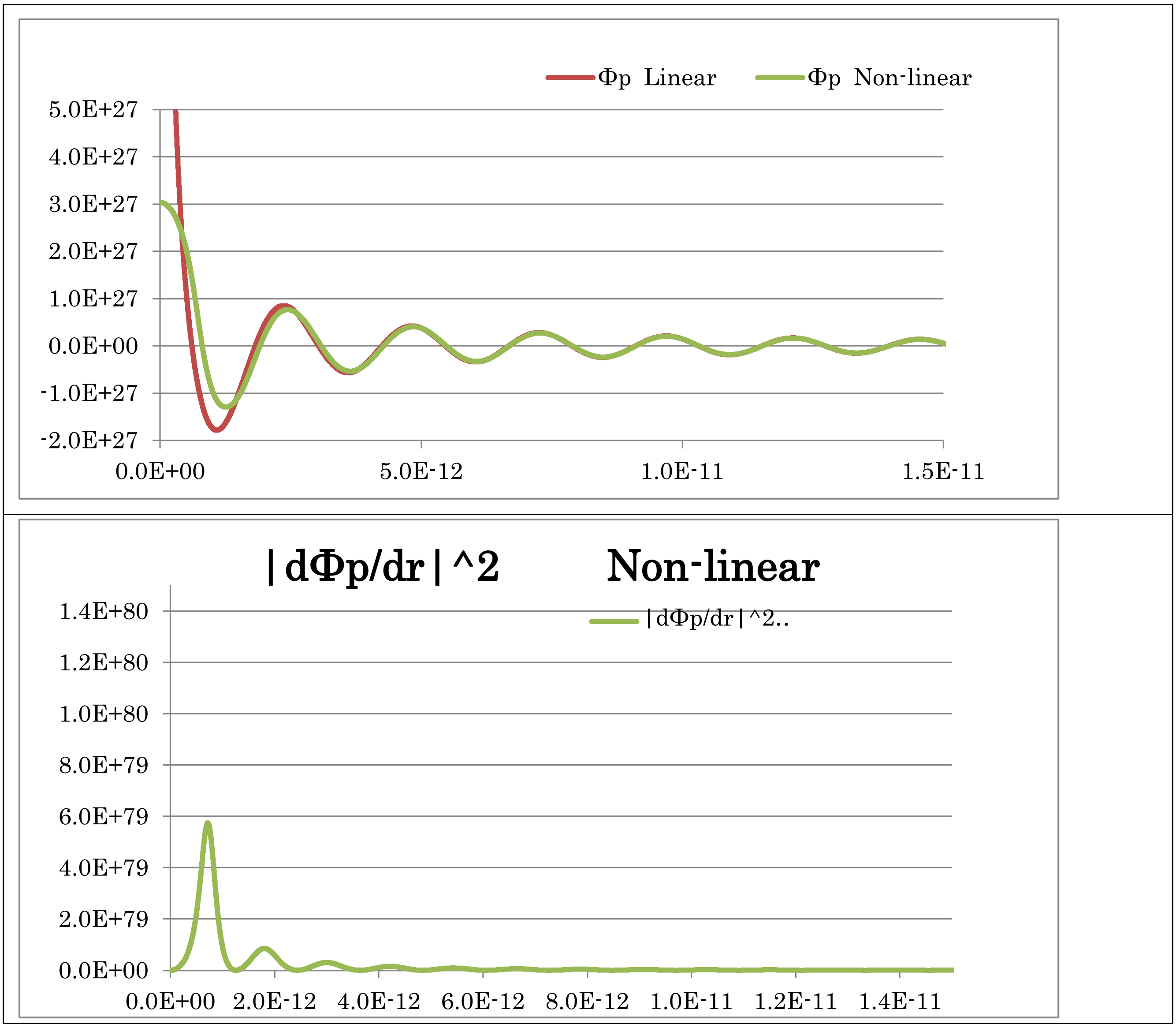
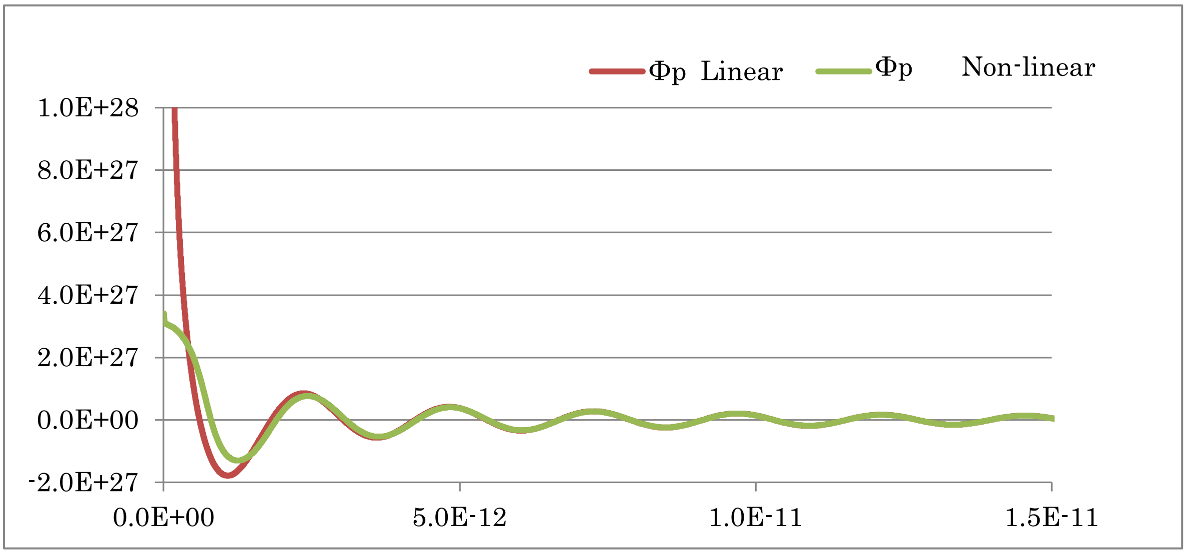


Figure 1. Converged solution $k_r = 1.75$ $|\nabla \phi_n|^2 = 3.5 \times 10^{75}$

Radius numbers when approached zero was not possible to find a solution be adjusted B will diverge. (Fig. 2)

In the case of $|\nabla \phi_n|^2 = 0$, When radius approaches zero it was not possible to find a solution because numeric values will diverge. (Fig. 2)

From this matter we guess that random noise ϕ_n has a significant influence on development and the life of the particle.



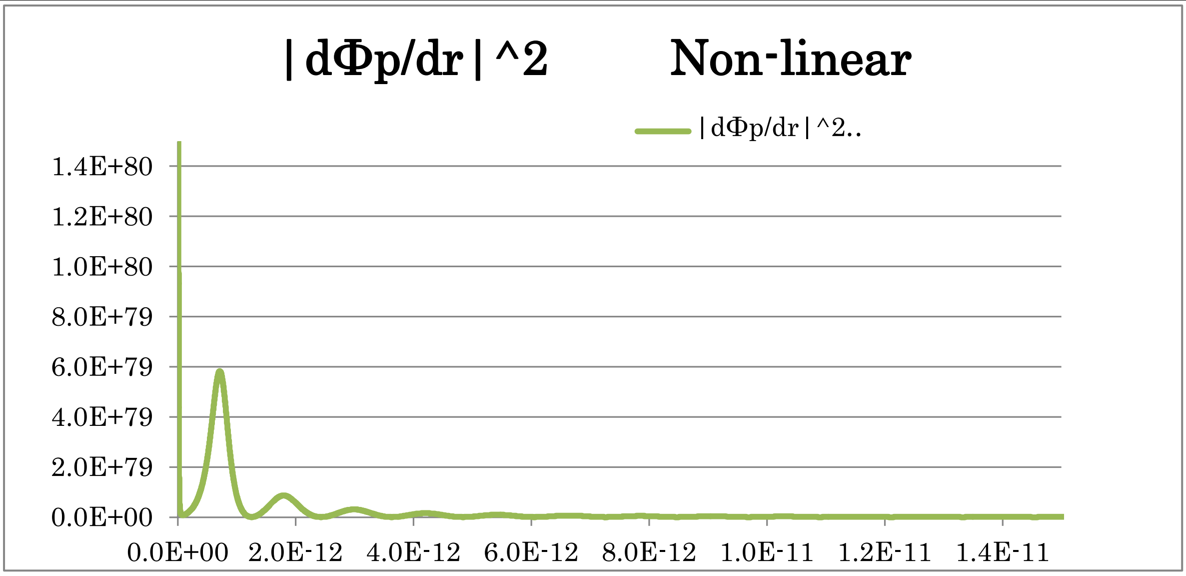


Figure 2 non-convergence solution $k_r = 1.75$ $|\nabla \phi_n|^2 = 0$

５． Conclusion

Nonlinearity is generated in the strong electromagnetic field.
Because Ricci tensor of the general relativity theory is depends on the electromagnetic energy.

We found a nonlinear standing wave solution in a particular condition of the strong electromagnetic field.
In this solution, the noise of electromagnetic field is an important factor.

6.Future Considerations

Now, I am considering the possibility that this standing wave solution obtained here is the elementary

particle.

Here was a standing wave solution for the sake of simplicity, but I am looking for also a traveling wave solution towards the outside from the center.
It seems to be random noise also occurs in the vicinity of the plurality of particles group.
It is possible that it also has frequency spectrum rather than complete random. And it is also likely to be filled in the whole universe.
Newly generated particle is affected by this spectrum of the noise.
If the eigen frequency and the charge of the particles is determined once in the early universe stage,
The eigen frequency and the charge of the subsequent generation particles are also considered to be likely to be limited thereto.
The gradient of the random noise is formed around the larger particles. So there it is also likely to be a reality of the gravitational field.
It has an energy density in this random noise itself.　It can become a source of gravitational field.
But it is on average distributed to the entire universe. Therefore it isn’t observed because gravity generated by it is canceled by each other.
When If taking the radius range of infinity, the sum of the electromagnetic energy of the standing wave becomes infinite.
In the area far away from the particle, synchronization signal is disturbed by noise from other particles around.
Therefore the mass is considered to be limited.