A standing wave solution of non-linear wave equation

[A condition when the frequency is in proportion to the weight]

4th November,2012

Tetsuya Nagai

1. Introduction

From the quantum mechanics, already we know the energy of particle is

$$E = h\nu$$

Where E is the energy, h is the Plank's constant, ν is the frequency.

From the special relativity

$$E = mc^2$$

Where m is the weight, c is the speed of light.

From two formulas in above we can get

$$m = \frac{h\nu}{c^2}$$

Therefore the weight is proportional to the frequency.

From the Relationship between light speed and weight,(see[1]) I expect that the weight is

proportional to the volume integral of $\nabla^2 c^2$

If there is a electro-magnetic standing wave around a charged particle and if we have a non-linear solution that's volume of integral of $\nabla^2 c^2$ is proportional to the standing wave's frequency then we can get a possibility of explanation about the fact that the weight is proportional to the frequency.

2. Standing wave

I consider about a electro-magnetic standing wave solution as below.

$$E = f(x, y, z)\cos(\omega t)\cos(\frac{\omega}{c_0} x) \quad (1)$$

From here E is the strength of electric field.
$f(x,y,z)$ is a function of (x,y,z)
ω is the specific rotation frequency.

c_0 is a constant that is called the light speed in the vacuum in general.
c is true light speed in vacuum.
c changes depend on the strength of electric field although this change is lower level than today's measurement accuracy.

As a non-linear factor I assume that

$$c^2 = c_0^2 - k_c E^p \qquad (2)$$

$$c_0^2 >> \left| k_c E^p \right| \qquad (2a)$$

Where k_c, p are constants.
In addition following wave formula is applicable.

$$\frac{\partial^2 E}{\partial t^2} = c^2 \nabla^2 E \qquad (3)$$

From(1)

$$\frac{\partial^2 E}{\partial t^2} = \frac{\partial}{\partial t}\left(f(x,y,z)(-\omega)\sin(\omega t)\cos(\frac{\omega}{c_0}x) \right)$$

$$= f(x,y,z)(-\omega^2)\cos(\omega t)\cos(\frac{\omega}{c_0}x) \qquad (4)$$

Also from (1)

$$c^2\nabla^2 E = c^2 \frac{\partial}{\partial x}\left(\frac{\partial f(x,y,z)}{\partial x}\cos(\omega t)\cos(\frac{\omega}{c_0}x) + f(x,y,z)\cos(\omega t)(-\frac{\omega}{c_0})\sin(\frac{\omega}{c_0}x) \right)$$

$$+ c^2\left(\frac{\partial^2 f(x,y,z)}{\partial y^2}\cos(\omega t)\cos(\frac{\omega}{c_0}x) \right) + c^2\left(\frac{\partial^2 f(x,y,z)}{\partial z^2}\cos(\omega t)\cos(\frac{\omega}{c_0}x) \right)$$

$$= c^2 \begin{pmatrix} \dfrac{\partial^2 f(x,y,z)}{\partial x^2} \cos(\omega t) \cos(\dfrac{\omega}{c_0} x) \\ + 2 \dfrac{\partial f(x,y,z)}{\partial x} \cos(\omega t)(-\dfrac{\omega}{c_0}) \sin(\dfrac{\omega}{c_0} x) + f(x,y,z) \cos(\omega t)(-\dfrac{\omega^2}{c^2{}_0}) \cos(\dfrac{\omega}{c_0} x) \end{pmatrix}$$

$$+ c^2 \left(\frac{\partial^2 f(x,y,z)}{\partial y^2} \cos(\omega t) \cos(\frac{\omega}{c_0} x) \right) + c^2 \left(\frac{\partial^2 f(x,y,z)}{\partial z^2} \cos(\omega t) \cos(\frac{\omega}{c_0} x) \right)$$

$$= c^2 \begin{pmatrix} \nabla^2 f(x,y,z) \cos(\omega t) \cos(\dfrac{\omega}{c_0} x) \\ + 2 \dfrac{\partial f(x,y,z)}{\partial x} \cos(\omega t)(-\dfrac{\omega}{c_0}) \sin(\dfrac{\omega}{c_0} x) \\ + f(x,y,z) \cos(\omega t)(-\dfrac{\omega^2}{c^2{}_0}) \cos(\dfrac{\omega}{c_0} x) \end{pmatrix} \qquad (5)$$

Using the fourier's transform at domain t and x I get the coefficient about

$\cos(\omega t) \cos(\frac{\omega}{c_0} x)$

From (4)

$$\frac{\omega}{\pi} \int_0^{\frac{2\pi}{\omega}} \frac{\omega}{\pi c_0} \int_0^{\frac{2\pi c_0}{\omega}} \frac{\partial^2 E}{\partial t^2} \cos(\omega t) \cos(\frac{\omega}{c_0} x) dx dt$$

$$= \frac{\omega}{\pi} \int_0^{\frac{2\pi}{\omega}} \frac{\omega}{\pi c_0} \int_0^{\frac{2\pi c_0}{\omega}} f(x,y,z)(-\omega^2) \cos^2(\omega t) \cos^2(\frac{\omega}{c_0} x) dx dt = -\omega^2 f(x,y,z) \qquad (6)$$

From(2)(5)

$$\frac{\omega}{\pi} \int_0^{\frac{2\pi}{\omega}} \frac{\omega}{\pi c_0} \int_0^{\frac{2\pi c_0}{\omega}} c^2 \nabla^2 E \cos(\omega t) \cos(\frac{\omega}{c_0} x) dx dt$$

$$= \frac{\omega}{\pi} \int_0^{\frac{2\pi}{\omega}} \frac{\omega}{\pi c_0} \int_0^{\frac{2\pi c_0}{\omega}} \left(c_0^2 - k_c E^p\right) \begin{pmatrix} \nabla^2 f(x,y,z) \cos^2(\omega t) \cos^2(\dfrac{\omega}{c_0} x) \\ + f(x,y,z) \cos(\omega t)^2 (-\dfrac{\omega^2}{c^2{}_0}) \cos^2(\dfrac{\omega}{c_0} x) \end{pmatrix} dx dt$$

$$= \frac{\omega}{\pi}\int_0^{\frac{2\pi}{\omega}} \frac{\omega}{\pi c_0}\int_0^{\frac{2\pi c_0}{\omega}} \begin{pmatrix} c_0^2\nabla^2 f(x,y,z)\cos^2(\omega t)\cos^2(\frac{\omega}{c_0}x) \\ + f(x,y,z)\cos(\omega t)^2(-\frac{\omega^2}{c^2{}_0})\cos^2(\frac{\omega}{c_0}x) \end{pmatrix} dxdt$$

$$+\frac{\omega}{\pi}\int_0^{\frac{2\pi}{\omega}} \frac{\omega}{\pi c_0}\int_0^{\frac{2\pi c_0}{\omega}} \left(-k_c f^p(x,y,z)\cos^p(\omega t)\cos^p(\frac{\omega}{c_0}x)\right)\left(f(x,y,z)\cos^2(\omega t)(-\frac{\omega^2}{c^2{}_0})\cos^2(\frac{\omega}{c_0}x)\right)dxdt$$

$$= c_0^2\nabla^2 f(x,y,z) - \omega^2 f(x,y,z) - k_c f^{p+1}(x,y,z)(-\frac{\omega^2}{c^2{}_0})\left(\frac{1}{\pi}\int_0^{2\pi}\cos^{p+2}(\theta)d\theta\right)^2 \quad (7)$$

Therefore

$$\nabla^2 f(x,y,z) = -k_c\frac{\omega^2}{c_0{}^4}\left(\frac{1}{\pi}\int_0^{2\pi}\cos^{p+2}(\theta)d\theta\right)^2 f^{p+1}(x,y,z) \quad (8)$$

$$k_L = k_c\frac{1}{c_0{}^4}\left(\frac{1}{\pi}\int_0^{2\pi}\cos^{p+2}(\theta)d\theta\right)^2 \quad (9)$$

Where k_L is a constant

From (8)(9)

$$\nabla^2 f(x,y,z) = -k_L\omega^2 f^{p+1}(x,y,z) \quad (10)$$

Therefore the laplacian of $f(x,y,z)$ is proportional to the product of the square of the frequency and $p+1$-th power of $f(x,y,z)$.

3. Normalization

I try to normalize the $f\ (x, y, z)$ with the radius and the angular frequency.

$$r_0 = \frac{1}{\omega^n} \quad (11)$$

Where r_0 is the standard radius.

$$R = \frac{r}{r_0} \quad (12)$$

Where R is the normalized radius.

From here I describe $f\ (r, \omega) = f\ (x, y, z, \omega)$

where $r = \sqrt{x^2 + y^2 + z^3}$ (13)

If $F\ (R)$ is normalized $f\ (r, \omega)$

$$F(R) = \frac{f\ (r, \omega)}{\omega^l} \quad (14)$$

Then

$$\nabla^2 f(r, \omega) = \frac{2}{r}\frac{\partial f(r, \omega)}{\partial r} + \frac{\partial^2 f(r, \omega)}{\partial r^2} = \frac{2\omega^n}{R}\omega^n \frac{\partial\left(\omega^l F(R)\right)}{\partial R} + \omega^{2n}\frac{\partial^2\left(\omega^l F(R)\right)}{\partial R^2}$$

$$= 2\omega^{2n+l}\frac{\partial F(R)}{\partial R} + \omega^{2n+1}\frac{\partial^2 F(R)}{\partial R^2} \quad (15)$$

From (10)(14)

$$\nabla^2 f(r, \omega) = -k_L \omega^2 f^{p+1}(r, \omega) = -k_L \omega^2 \omega^{pl+l} F^{p+1}(R) = -k_L \omega^{pl+l+2} F^{p+1}(R) \quad (16)$$

From(15)(16)

$$-k_L \omega^{pl+l+2} F^{p+1}(R) = 2\omega^{2n+l} \frac{\partial F(R)}{\partial R} + \omega^{2n+l} \frac{\partial^2 F(R)}{\partial R^2} \quad (17)$$

If (17) does not depends on ω then following condition is necessary.

$$pl + l + 2 = 2n + l \quad (18)$$

Therefore

$$pl + 2 = 2n \quad (18a)$$

Then I devide the (17)by $\omega^{pl+l+2} = \omega^{2n+l}$

$$-k_L F^{p+1}(R) = 2\frac{\partial F(R)}{\partial R} + \frac{\partial^2 F(R)}{\partial R^2} \quad (19)$$

Finally we get normalized formula that does not depends on ω.

4. Estimation of total weight

Next challenge is to demand the total weight.

I describe (2) again.

$$c^2 = c_0^2 - k_c E^p \quad (2)$$

If $\overline{c^2}$ is the average of c^2.

$$\overline{c^2} = \frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} \frac{\omega}{2\pi c_0}\int_0^{\frac{2\pi c_0}{\omega}} c^2 dxdt = \frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} \frac{\omega}{2\pi c_0}\int_0^{\frac{2\pi c_0}{\omega}} \left(c_0^2 - k_c E^p\right) dxdt$$

$$= \frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} \frac{\omega}{2\pi c_0}\int_0^{\frac{2\pi c_0}{\omega}} \left(c_0^2 - k_c f^p(r,\omega)\cos^p(\omega t)\cos^p\left(\frac{\omega}{c_0}x\right)\right) dxdt$$

$$= c_0^2 - k_c \left(\frac{1}{2\pi}\int_0^{2\pi}\cos^p(\theta)d\theta\right)^2 f^p(r,\omega) \quad (20)$$

$$\nabla^2 \overline{c^2} = -k_c \left(\frac{1}{2\pi}\int_0^{2\pi}\cos^p(\theta)d\theta\right)^2 \nabla^2 f^p(r,\omega) \quad (20a)$$

$$\nabla^2 \overline{c^2} = -k_c \left(\frac{1}{2\pi}\int_0^{2\pi}\cos^p(\theta)d\theta\right)^2 \left(\frac{2}{r}\frac{\partial f^p(r,\omega)}{\partial r} + \frac{\partial^2 f^p(r,\omega)}{\partial r^2}\right)$$

$$= -k_c \left(\frac{1}{2\pi}\int_0^{2\pi}\cos^p(\theta)d\theta\right)^2 \left(\frac{2\omega^n}{R}\omega^n \frac{\partial\left(\omega^{pl} F^{pl}(R)\right)}{\partial R} + \omega^{2n}\frac{\partial^2\left(\omega^{pl} F^{pl}(R)\right)}{\partial R^2}\right)$$

$$= -k_c \left(\frac{1}{2\pi}\int_0^{2\pi}\cos^p(\theta)d\theta\right)^2 \left(\frac{2\omega^{2n+pl}}{R}\frac{\partial\left(F^{pl}(R)\right)}{\partial R} + \omega^{2n+pl}\frac{\partial^2\left(F^{pl}(R)\right)}{\partial R^2}\right) \quad (21)$$

From Relationship between light speed and weight (see[1])

$$m = \frac{1}{16\pi G}\int_v \nabla^2 c^2 dv \qquad (22)$$

where m is the weight,

dv is differential volume.

From (21)(22)

$$m = \frac{1}{16\pi G}\int_v \nabla^2 c^2 dv = \frac{1}{16\pi G}\int_{R=0}^{\infty}\left(\nabla^2 \overline{c^2}\right)\pi r^2 dr \qquad (22a)$$

$$m = -\frac{1}{16\pi G}k_c\left(\frac{1}{2\pi}\int_0^{2\pi}\cos^p(\theta)d\theta\right)^2\int_{R=0}^{\infty}\left(\frac{2\omega^{2n+pl}}{R}\frac{\partial\left(F^{pl}(R)\right)}{\partial R}+\omega^{2n+pl}\frac{\partial^2\left(F^{pl}(R)\right)}{\partial R^2}\right)\pi\frac{R^2}{\omega^{2n}}\frac{1}{\omega^n}dR$$

$$= -\frac{1}{16\pi G}k_c\left(\frac{1}{2\pi}\int_0^{2\pi}\cos^p(\theta)d\theta\right)^2\omega^{pl-n}\int_{R=0}^{\infty}\left(\frac{2}{R}\frac{\partial\left(F^{pl}(R)\right)}{\partial R}+\frac{\partial^2\left(F^{pl}(R)\right)}{\partial R^2}\right)\pi R^2 dR$$

(23)

If the frequency is proportional to the weight then

$$pl - n = 1 \qquad (24)$$

Therefore

$$m = k_1\omega = 2\pi k_1\nu \qquad (25)$$

where ν is the frequency.

k_1 is a constant as

$$k_1 = -\frac{1}{16\pi G}k_c\left(\frac{1}{2\pi}\int_0^{2\pi}\cos^p(\theta)d\theta\right)^2\int_{R=0}^{\infty}\left(\frac{2}{R}\frac{\partial\left(F^{pl}(R)\right)}{\partial R}+\frac{\partial^2\left(F^{pl}(R)\right)}{\partial R^2}\right)\pi R^2 dR$$

(26)

I describe the conditions again below

$$pl + 2 = 2n \qquad (18a)$$

$$pl - n = 1 \qquad (24)$$

From those conditions

$n = 3$ (27)

$pl = 4$ (28)

I try to verify a simple case that the coefficients p and l are positive integer.

From (9)(10)

$$k_L = k_c \frac{1}{{c_0}^4}\left(\frac{1}{\pi}\int_0^{2\pi}\cos^{p+2}(\theta)d\theta\right)^2 = k_c \frac{1}{{c_0}^4}\left(\frac{2(p+1)!!}{(p+2)!!}\right)^2 \qquad (29)$$

$$\nabla^2 f(x,y,z) = -k_L \omega^2 f^{p+1}(x,y,z) = -k_c \frac{1}{{c_0}^4}\left(\frac{2(p+1)!!}{(p+2)!!}\right)^2 \omega^2 f^{p+1}(x,y,z) \qquad (29a)$$

From(25)(26)

$$m = k_1\omega = -k_m k_c\left(\frac{1}{2\pi}\int_0^{2\pi}\cos^p(\theta)d\theta\right)^2 \omega\int_{R=0}^{\infty}\left(\frac{2}{R}\frac{\partial\left(F^{pl}(R)\right)}{\partial R} + \frac{\partial^2\left(F^{pl}(R)\right)}{\partial R^2}\right)\pi R^2 dR$$

$$= -\frac{1}{16\pi G}k_c\left(\frac{(p-1)!!}{p!!}\right)^2 \omega\int_{R=0}^{\infty}\left(\frac{2}{R}\frac{\partial\left(F^{pl}(R)\right)}{\partial R} + \frac{\partial^2\left(F^{pl}(R)\right)}{\partial R^2}\right)\pi R^2 dR \qquad (30)$$

5. Calculation

From (28) if p is certain then l will be certain.

Therefore

In case of p is not determined	$pl = 4$ (28) $c^2 = c_0^2 - k_c E^p$ (2) $r_0 = \frac{1}{\omega^n} = \frac{1}{\omega^3}$ (11) $F(R) = \frac{f\ (r,\omega)}{\omega^l}$ (14)
In case of $p = 1$	$l = 4$ (31) $c^2 = c_0^2 - k_c E$ (32) $r_0 = \frac{1}{\omega^3}$ (33) $F(R) = \frac{f\ (r,\omega)}{\omega^4}$ (34)
In case of $p = 2$	$l = 2$ (35) $c^2 = c_0^2 - k_c E^2$ (36) $r_0 = \frac{1}{\omega^3}$ (37) $F(R) = \frac{f\ (r,\omega)}{\omega^2}$ (38)
In case of $p = 4$	$l = 1$ (39) $c^2 = c_0^2 - k_c E^4$ (40) $r_0 = \frac{1}{\omega^3}$ (41) $F(R) = \frac{f\ (r,\omega)}{\omega^1}$ (42)

I try to verify the form of function of $f\ (r,\omega)$.

From(10)

$$\frac{\partial f(r,\omega)}{\partial r}=\int_0^r\frac{\partial^2 f(r,\omega)}{\partial r^2}dr+\left.\frac{\partial f(r,\omega)}{\partial r}\right|_{r=0}=\int_0^r\left[\nabla^2 f(r,\omega)-\frac{2}{r}\frac{\partial f(r,\omega)}{\partial r}\right]dr+\left.\frac{\partial f(r,\omega)}{\partial r}\right|_{r=0}$$

$$=\int_0^r\left[-k_L\omega^2 f^{p+1}(x,y,z)-\frac{2}{r}\frac{\partial f(r,\omega)}{\partial r}\right]dr+\left.\frac{\partial f(r,\omega)}{\partial r}\right|_{r=0}$$

$$f(r,\omega)=\int_0^r\frac{\partial f(r,\omega)}{\partial r}dr+f(r,\omega)\big|_{r=0}$$

Therefore we can get $f\ (r,\omega)$ by integration from the center to outer.

In a simplest case as below

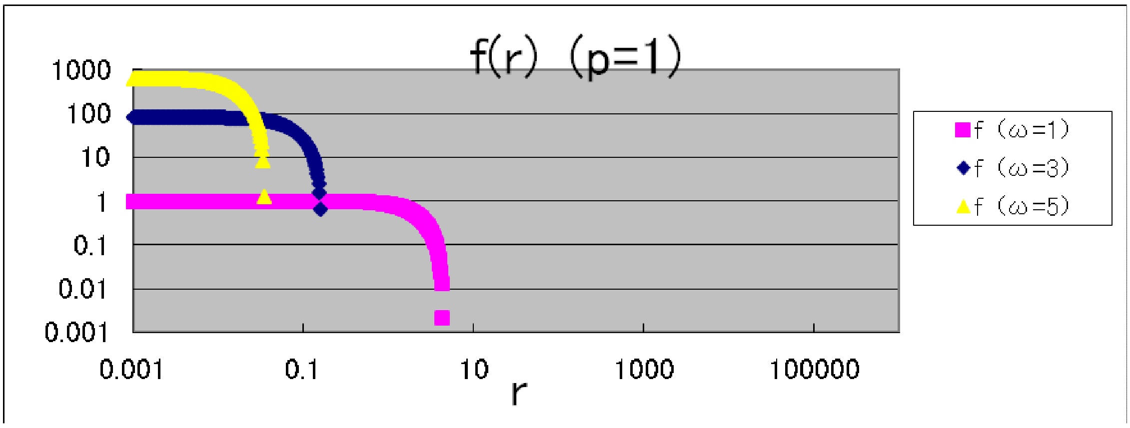
$$f(r,\omega)\big|_{r=0}=1$$

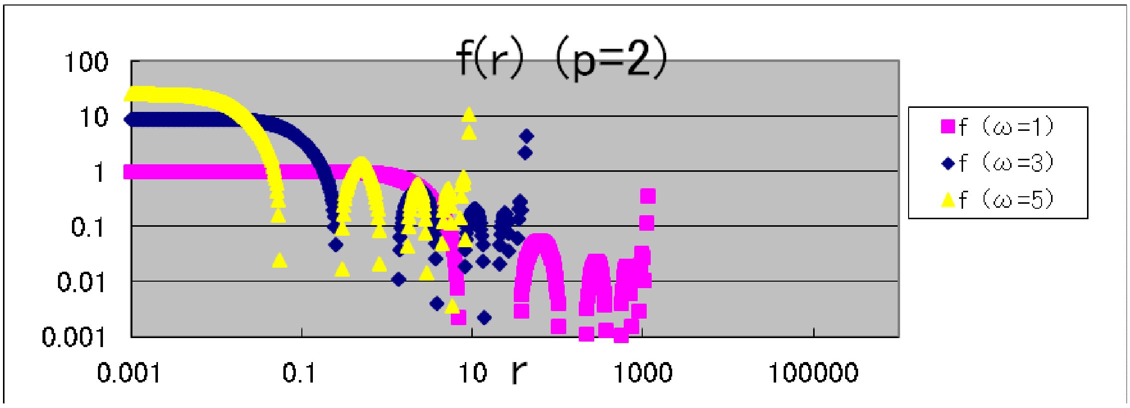
$$\left.\frac{\partial f(r,\omega)}{\partial r}\right|_{r=0}=0$$

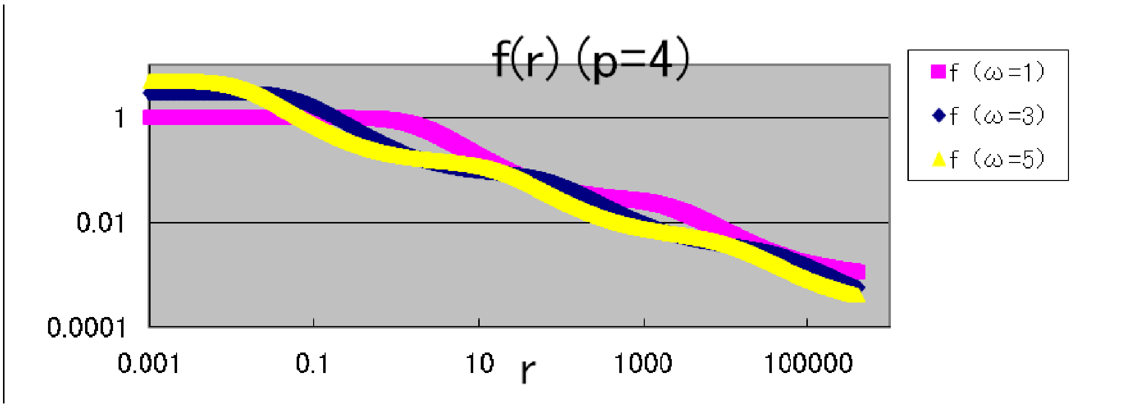
$$k_L=1$$

I calculate $f\ (r,\omega)$ using difference method.

I plot a double logarithmic chart that’s horizontal axis is r .



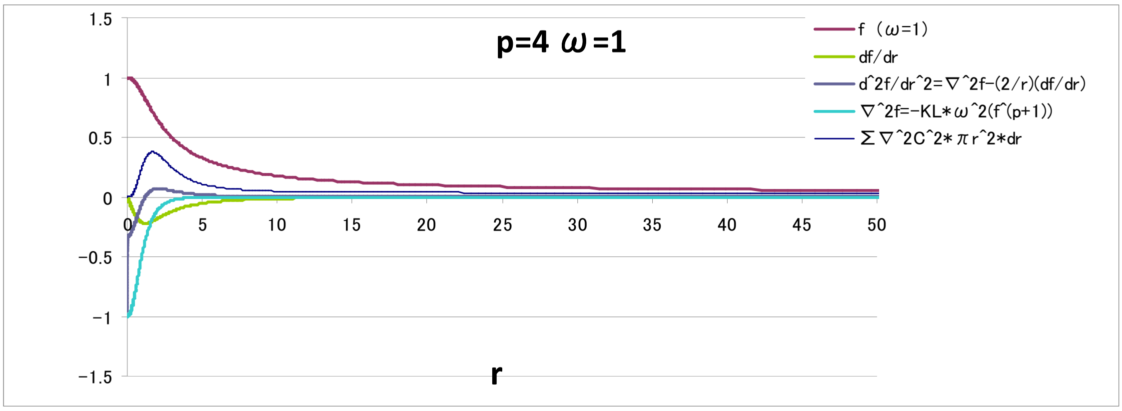




In the case of p= 1 , p=2 it collapses with the increase of r.

Although in the case of p=4 it does not collapse.

In case of $k_c = 1, p = 4, \omega = 1$ all values are converge in linear scale as following.



I decided I will verify the case of $p = 4$ mainly.

$$p = 4 \qquad (42a)$$

From(42a) (20a)

$$\nabla^2 \overline{c^2} = -k_c \left(\frac{1}{2\pi} \int_0^{2\pi} \cos^p(\theta) d\theta \right)^2 \nabla^2 f^p(r,\omega)$$

$$= -k_c \left(\frac{(p-1)!!}{p!!} \right)^2 \nabla^2 f^p(r,\omega)$$

$$= -k_c \left(\frac{9}{64} \right) \nabla^2 f^4(r,\omega) \qquad (43)$$

From(29a)

$$\nabla^2 f(x,y,z) = -k_c \frac{1}{{c_0}^4} \left(\frac{2(p+1)!!}{(p+2)!!} \right)^2 \omega^2 f^{p+1}(x,y,z) = -k_c \frac{1}{{c_0}^4} \frac{25}{64} \omega^2 f^{p+1}(x,y,z)$$

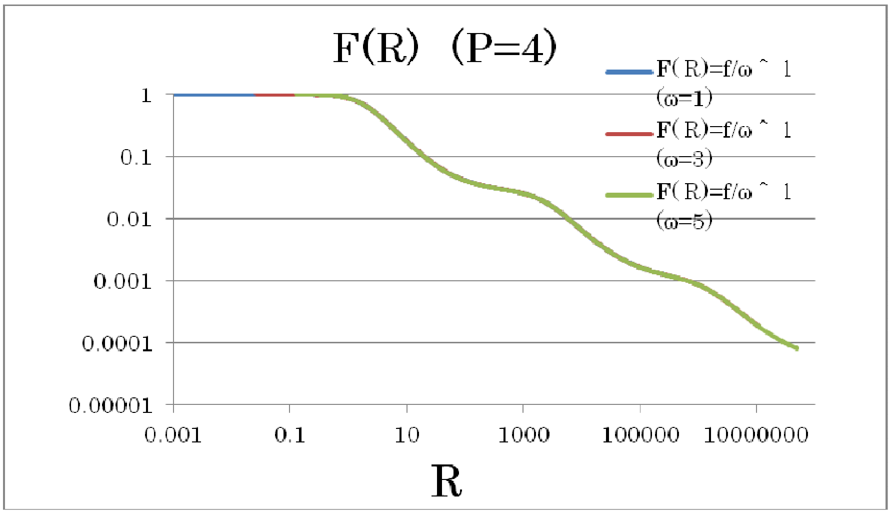
(44)

From(30)

$$m = k_1\omega = -\frac{1}{16\pi G}k_c\left(\frac{(p-1)!!}{p!!}\right)^2\omega\int_{R=0}^{\infty}\left(\frac{2}{R}\frac{\partial\left(F^{pl}(R)\right)}{\partial R}+\frac{\partial^2\left(F^{pl}(R)\right)}{\partial R^2}\right)\pi R^2 dR$$
$$= -\frac{1}{16\pi G}k_c\frac{9}{64}\omega\int_{R=0}^{\infty}\left(\frac{2}{R}\frac{\partial\left(F^4(R)\right)}{\partial R}+\frac{\partial^2\left(F^4(R)\right)}{\partial R^2}\right)\pi R^2 dR$$

(45)

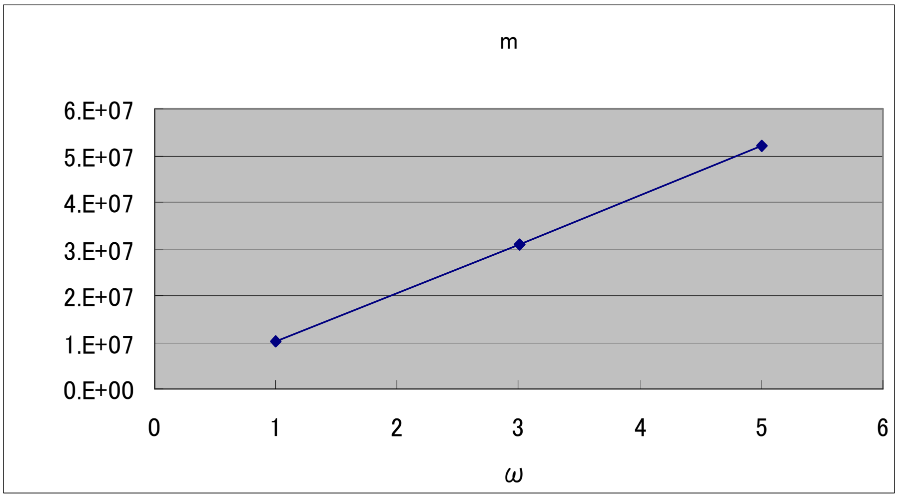
In the case of $p = 4$ from(42) I calculate $F(R)$.



I confirmed that $F(R)$ does not change when ω changes .

The normalization is successful.

I calculate the weight m from (21) (22a).



I confirmed that m changes depends on ω.

The weight is proportional to frequency certainly.

6. Conclusion

I found a electro-magnetic non-linear standing wave solution that's volume of integral of $\nabla^2 c^2$ is proportional to the standing wave's frequency in a specific condition.
From the Relationship between light speed and weight,(see[1]) I expect that the weight is

proportional to the volume integral of $\nabla^2 c^2$.

Thus the weight is proportional to frequency in the condition.
As a result we have a new possibility of elucidation about the essential connection between the quantum mechanics and relativity.

References

1. Tetsuya Nagai "Relationship between light speed and weight"
   http://www.tegakinet.jp/wave/waveE.files/lightweightE.pdf