定在波からのシュレーディンガー方程式の導出

2012/8/15 修正

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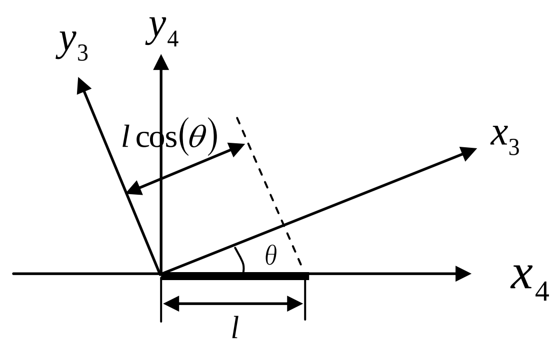
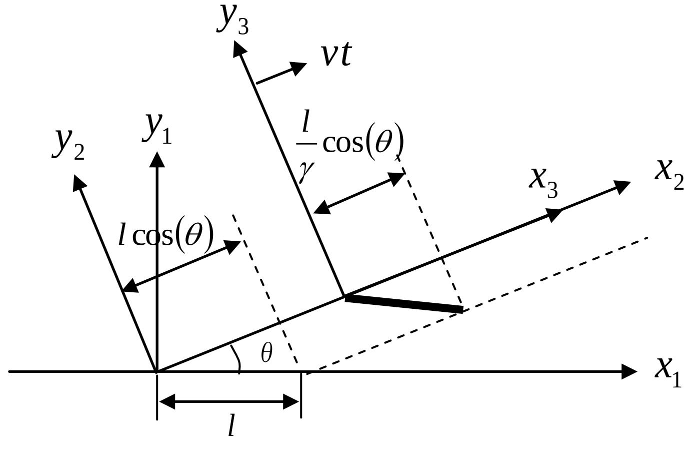
θ の方向に向かって速度 v で移動する系を考える。
この系の中に非線形波動方程式の定在波解があるとする。
定在波をこの系から見たときは以下のようになる。
ただし l は一波長分の長さ



定在波を静止系から見ると以下のようになる。



$$x_2 = x_1 \cos(\theta) + y_1 \sin(\theta)$$
$$y_2 = -x_1 \sin(\theta) + y_1 \cos(\theta)$$

$$x_4 = x_3 \cos(\theta) - y_3 \sin(\theta)$$
$$y_4 = x_3 \sin(\theta) + y_3 \cos(\theta)$$

ローレンツ変換により

$$x_3 = \gamma(x_2 - vt)$$
$$y_3 = y_2$$
$$t' = \gamma\left(t - \frac{vx_2}{c^2}\right)$$

ただし

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$c >> |v|$$

E_1, E_2 を定在波とし

$$E_1 = f(x_1, y_1)\cos(\omega t)\cos\left(\frac{\omega x_1}{c}\right)$$
$$E_2 = f(x_4, y_4)\cos(\omega t')\cos\left(\frac{\omega x_4}{c}\right)$$

とするがここでは近似的に

$f(x_1, y_1) = f_1, f(x_4, y_4) = f_2$　　f_1, f_2　はともに定数とし

$$E_1 = f_1 \cos(\omega t)\cos\left(\frac{\omega x_4}{c}\right)$$
$$E_2 = f_2 \cos(\omega t')\cos\left(\frac{\omega x_4}{c}\right)$$

とする。

定在波を進行波と後退波に分けて

$$E_{1+} = \frac{1}{2}\cos\left(\omega t - \frac{\omega x_1}{c}\right)$$

$$E_{1-} = \frac{1}{2}\cos\left(\omega t + \frac{\omega x_1}{c}\right)$$

$$E_{2+} = \frac{1}{2}\cos\left(\omega t' - \frac{\omega x_4}{c}\right)$$

$$E_{2-} = \frac{1}{2}\cos\left(\omega t' + \frac{\omega x_4}{c}\right)$$

とすると

$$E_1 = \cos(\omega t)\cos\left(\frac{\omega x_1}{c}\right) = \frac{1}{2}\left(\cos\left(\omega t - \frac{\omega x_1}{c}\right) + \cos\left(\omega t + \frac{\omega x_1}{c}\right)\right)$$

$$= E_{1+} + E_{1-}$$

$$E_2 = \cos(\omega t')\cos\left(\frac{\omega x_4}{c}\right) = \frac{1}{2}\left(\cos\left(\omega t' - \frac{\omega x_4}{c}\right) + \cos\left(\omega t' + \frac{\omega x_4}{c}\right)\right)$$

$$= E_{2+} + E_{2-}$$

$$E_{1+} = \frac{1}{2}\cos\left(\omega t - \frac{\omega x_1}{c}\right) = \frac{1}{2}\cos\left(\omega(t) - \frac{\omega}{c}(x_2\cos(\theta) - y_2\sin(\theta))\right)$$

$$E_{1-} = \frac{1}{2}\cos\left(\omega t + \frac{\omega x_1}{c}\right) = \frac{1}{2}\cos\left(\omega(t) + \frac{\omega}{c}(x_2\cos(\theta) - y_2\sin(\theta))\right)$$

$$E_{2+} = \frac{1}{2}\cos\left(\omega t' - \frac{\omega x_4}{c}\right) = \frac{1}{2}\cos\left(\omega\gamma\left(t - \frac{v x_2}{c^2}\right) - \frac{\omega}{c}(x_3\cos(\theta) - y_3\sin(\theta))\right)$$

$$= \frac{1}{2}\cos\left(\omega\gamma\left(t - \frac{v x_2}{c^2}\right) - \frac{\omega}{c}(\gamma(x_2 - vt)\cos(\theta) - y_3\sin(\theta))\right)$$

$$= \frac{1}{2}\cos\left(\omega\gamma t\left(1 + \frac{v}{c}\cos(\theta)\right) - \frac{\omega\gamma x_2}{c}\left(\cos(\theta) + \frac{v}{c}\right) + \frac{\omega y_2}{c}\sin(\theta)\right)$$

となる

$$\omega_2 = \omega\gamma\left(1+\frac{v}{c}\cos(\theta)\right)$$

$$\cos\theta_2 = \frac{\cos\theta + \frac{v}{c}}{1+\frac{v}{c}\cos\theta} \qquad \sin\theta_2 = \frac{-\frac{1}{\gamma}\sin\theta}{1+\frac{v}{c}\cos\theta}$$

とすると

$$E_{2+} = \frac{1}{2}\cos\left(\omega_2\left(t-\frac{1}{c}(x_2\cos(\theta_2)+y_2\sin(\theta_2))\right)\right)$$

となる

$$E_{2-} = \frac{1}{2}\cos\left(\omega t' + \frac{\omega x_4}{c}\right) = \frac{1}{2}\cos\left(\omega\gamma\left(t-\frac{vx_2}{c}\right)+\frac{\omega}{c}(x_3\cos(\theta)-y_3\sin(\theta))\right)$$

$$= \frac{1}{2}\cos\left(\omega\gamma\left(t-\frac{vx_2}{c^2}\right)+\frac{\omega}{c}(\gamma(x_2-vt)\cos(\theta)-y_3\sin(\theta))\right)$$

$$= \frac{1}{2}\cos\left(\omega\gamma t\left(1-\frac{v}{c}\cos(\theta)\right)-\frac{\omega\gamma x_2}{c}\left(-\cos(\theta)+\frac{v}{c}\right)-\frac{\omega y_2}{c}\sin(\theta)\right)$$

$$\omega_1 = \omega\gamma\left(1-\frac{v}{c}\cos(\theta)\right)$$

$$\cos\theta_1 = \frac{\cos\theta - \frac{v}{c}}{1-\frac{v}{c}\cos\theta}$$

$$\sin\theta_1 = \frac{-\frac{1}{\gamma}\sin\theta}{1-\frac{v}{c}\cos\theta}$$

とすると

$$E_{2-} = \frac{1}{2}\cos\left(\omega_1\left(t+\frac{1}{c}(x_2\cos(\theta_1)+y_2\sin(\theta_1))\right)\right)$$

となる。

T_1 を時間の分解能とし

$$\frac{1}{\omega_2-\omega},\frac{1}{\omega-\omega_1}>>T_1>>\frac{1}{2\omega}$$ とする。

この分解能をもって二つの進行波成分の和の二乗を観測したものを Ψ_+

二つの後退波成分の和の二乗を観測したものを Ψ_- とすると

$$\Psi_+ \;=\frac{1}{T_1}\int_0^{T_1}(E_{1+}+E_{2+})^2dt$$

$$=\frac{1}{T_1}\int_0^{T_1}\left(\frac{1}{4}\cos^2\left(\omega(t)-\frac{\omega}{c}(x_2\cos(\theta)-y_2\sin(\theta))\right)\right)dt$$

$$+\frac{1}{T_1}\int_0^{T_1}\left(\frac{1}{4}\cos^2\left(\omega_2 t-\frac{\omega_2}{c}(x_2\cos(\theta_2)+y_2\sin(\theta_2))\right)\right)dt$$

$$+\frac{1}{T_1}\int_0^{T_1}\begin{pmatrix}\frac{1}{2}\cos\left(\omega(t)-\frac{\omega}{c}(x_2\cos(\theta)-y_2\sin(\theta))\right)\\ \frac{1}{2}\cos\left(\omega_2 t-\frac{\omega_2}{c}(x_2\cos(\theta_2)+y_2\sin(\theta_2))\right)\end{pmatrix}dt$$

$$=\frac{1}{T_1}\int_0^{T_1}\left(\frac{1}{8}+\frac{1}{8}\cos\left(2\left(\omega(t)-\frac{\omega}{c}(x_2\cos(\theta)-y_2\sin(\theta))\right)\right)\right)dt$$

$$+\frac{1}{T_1}\int_0^{T_1}\left(\frac{1}{8}+\frac{1}{8}\cos\;\left(2\left(\omega_2 t-\frac{\omega_2}{c}(x_2\cos(\theta_2)+y_2\sin(\theta_2))\right)\right)\right)dt$$

$$+\frac{1}{T_1}\int_0^{T_1}\begin{pmatrix}\frac{1}{4}\cos\begin{pmatrix}\left(\omega t-\frac{\omega x_2}{c}(\cos(\theta))+\frac{\omega y_2}{c}\sin(\theta)\right)\\ +\left(\omega_2 t-\frac{\omega_2}{c}(x_2\cos(\theta_2)+y_2\sin(\theta_2))\right)\end{pmatrix}\\ +\frac{1}{4}\cos\begin{pmatrix}\left(\omega t-\frac{\omega x_2}{c}(\cos(\theta))+\frac{\omega y_2}{c}\sin(\theta)\right)\\ -\left(\omega_2 t-\frac{\omega_2}{c}(x_2\cos(\theta_2)+y_2\sin(\theta_2))\right)\end{pmatrix}\end{pmatrix}dt$$

$$= \frac{1}{8} + \frac{1}{8}$$

$$+\frac{1}{T_1}\int_0^{T_1}\begin{pmatrix} \frac{1}{4}\cos\begin{pmatrix}(\omega+\omega_2)t \\ -\left(\frac{\omega}{c}(x_2(\cos(\theta))-y_2\sin(\theta))+\frac{\omega_2}{c}(x_2\cos(\theta_2)+y_2\sin(\theta_2))\right)\end{pmatrix} \\ +\frac{1}{4}\cos\begin{pmatrix}(\omega-\omega_2)t \\ -\left(\frac{\omega}{c}(x_2(\cos(\theta))-y_2\sin(\theta))-\frac{\omega_2}{c}(x_2\cos(\theta_2)+y_2\sin(\theta_2))\right)\end{pmatrix}\end{pmatrix}dt$$

$$= \frac{1}{4} + \frac{1}{4}\cos\begin{pmatrix}(\omega-\omega_2)t \\ -\left(\frac{\omega}{c}(x_2(\cos(\theta))-y_2\sin(\theta))-\frac{\omega_2}{c}(x_2\cos(\theta_2)+y_2\sin(\theta_2))\right)\end{pmatrix}$$

$$\Psi_- \;= \frac{1}{T_1}\int_0^{T_1}\left(E_{1-}+E_{2-}\right)^2 dt$$

$$= \frac{1}{T_1}\int_0^{T_1}\left(\frac{1}{4}\cos^2\left(\omega(t)+\frac{\omega}{c}(x_2\cos(\theta)-y_2\sin(\theta))\right)\right)dt$$

$$+\frac{1}{T_1}\int_0^{T_1}\left(\frac{1}{4}\cos^2\left(\omega_1 t+\frac{\omega_1}{c}(x_2\cos(\theta_1)+y_2\sin(\theta_1))\right)\right)dt$$

$$+\frac{1}{T_1}\int_0^{T_1}\begin{pmatrix}\frac{1}{2}\cos\left(\omega(t)+\frac{\omega}{c}(x_2\cos(\theta)-y_2\sin(\theta))\right) \\ \frac{1}{2}\cos\left(\omega_1 t+\frac{\omega_1}{c}(x_2\cos(\theta_1)+y_2\sin(\theta_1))\right)\end{pmatrix}dt$$

$$=\frac{1}{T_1}\int_0^{T_1}\left(\frac{1}{8}+\frac{1}{8}\cos\left(2\left(\omega(t)+\frac{\omega}{c}\left(x_2\cos(\theta)-y_2\sin(\theta)\right)\right)\right)\right)dt$$

$$+\frac{1}{T_1}\int_0^{T_1}\left(\frac{1}{8}+\frac{1}{8}\cos\ \left(2\left(\omega_1 t+\frac{\omega_1}{c}\left(x_2\cos(\theta_1)+y_2\sin(\theta_1)\right)\right)\right)\right)dt$$

$$+\frac{1}{T_1}\int_0^{T_1}\left(\begin{array}{l}\frac{1}{4}\cos\left(\begin{array}{l}\left(\omega t-\frac{\omega x_2}{c}(\cos(\theta))+\frac{\omega y_2}{c}\sin(\theta)\right)\\+\left(\omega_1 t+\frac{\omega_1}{c}\left(x_2\cos(\theta_1)+y_2\sin(\theta_1)\right)\right)\end{array}\right)\\+\frac{1}{4}\cos\left(\begin{array}{l}\left(\omega t-\frac{\omega x_2}{c}(\cos(\theta))+\frac{\omega y_2}{c}\sin(\theta)\right)\\-\left(\omega_1 t+\frac{\omega_1}{c}\left(x_2\cos(\theta_1)+y_2\sin(\theta_1)\right)\right)\end{array}\right)\end{array}\right)dt$$

$$=\frac{1}{8}+\frac{1}{8}$$

$$+\frac{1}{T_1}\int_0^{T_1}\left(\begin{array}{l}\frac{1}{4}\cos\left(\begin{array}{l}(\omega+\omega_1)t\\+\left(\frac{\omega}{c}\left(x_2(\cos(\theta))-y_2\sin(\theta)\right)+\frac{\omega_1}{c}\left(x_2\cos(\theta_1)+y_2\sin(\theta_1)\right)\right)\end{array}\right)\\+\frac{1}{4}\cos\left(\begin{array}{l}(\omega-\omega_1)t\\+\left(\frac{\omega}{c}\left(x_2(\cos(\theta))-y_2\sin(\theta)\right)-\frac{\omega_1}{c}\left(x_2\cos(\theta_1)+y_2\sin(\theta_1)\right)\right)\end{array}\right)\end{array}\right)dt$$

$$=\frac{1}{4}+\frac{1}{4}\cos\left(\begin{array}{l}(\omega-\omega_1)t\\+\left(\frac{\omega}{c}\left(x_2(\cos(\theta))-y_2\sin(\theta)\right)-\frac{\omega_1}{c}\left(x_2\cos(\theta_1)+y_2\sin(\theta_1)\right)\right)\end{array}\right)$$

二つの成分をまとめて記すと

$$\Psi_{+} = \frac{1}{4} + \frac{1}{4}\cos\begin{pmatrix} (\omega - \omega_2)t \\ -\left(\frac{\omega}{c}(x_2(\cos(\theta)) - y_2\sin(\theta)) - \frac{\omega_2}{c}(x_2\cos(\theta_2) + y_2\sin(\theta_2))\right) \end{pmatrix}$$

$$\Psi_{-} = \frac{1}{4} + \frac{1}{4}\cos\begin{pmatrix} (\omega - \omega_1)t \\ +\left(\frac{\omega}{c}(x_2(\cos(\theta)) - y_2\sin(\theta)) - \frac{\omega_1}{c}(x_2\cos(\theta_1) + y_2\sin(\theta_1))\right) \end{pmatrix}$$

T_2を時間の分解能とし

$\frac{1}{-2\omega + \omega_1 + \omega_2} >> T_2 >> \frac{1}{\omega_2 - \omega_1}$　とする。

この分解能をもって上記二つの成分の交流成分の和の二乗を観測したものを$\Psi_s{}^2$とすると

$$\Psi_s{}^2 = \frac{1}{T_2}\int_0^{T_2}\left(\Psi_{+} + \Psi_{-} - \frac{1}{2}\right)^2 dt$$

$$= \frac{1}{T_2}\int_0^{T_2}\begin{pmatrix} \frac{1}{4}\cos\begin{pmatrix} (\omega - \omega_2)t \\ -\left(\frac{\omega}{c}(x_2(\cos(\theta)) - y_2\sin(\theta)) - \frac{\omega_2}{c}(x_2\cos(\theta_2) + y_2\sin(\theta_2))\right) \end{pmatrix} \\ +\frac{1}{4}\cos\begin{pmatrix} (\omega - \omega_1)t \\ +\left(\frac{\omega}{c}(x_2(\cos(\theta)) - y_2\sin(\theta)) - \frac{\omega_1}{c}(x_2\cos(\theta_1) + y_2\sin(\theta_1))\right) \end{pmatrix} \end{pmatrix}^2 dt$$

$$=\frac{1}{T_2}\int_0^{T_2}\left(\begin{array}{l}\frac{1}{16}\cos^2\left(\begin{array}{l}(\omega-\omega_2)t\\-\left(\frac{\omega}{c}(x_2(\cos(\theta))-y_2\sin(\theta))-\frac{\omega_2}{c}(x_2\cos(\theta_2)+y_2\sin(\theta_2))\right)\end{array}\right)\\+\frac{1}{16}\cos^2\left(\begin{array}{l}(\omega-\omega_1)t\\+\left(\frac{\omega}{c}(x_2(\cos(\theta))-y_2\sin(\theta))-\frac{\omega_1}{c}(x_2\cos(\theta_1)+y_2\sin(\theta_1))\right)\end{array}\right)\\+\frac{1}{8}\cos\left(\begin{array}{l}(\omega-\omega_2)t\\-\left(\frac{\omega}{c}(x_2(\cos(\theta))-y_2\sin(\theta))-\frac{\omega_2}{c}(x_2\cos(\theta_2)+y_2\sin(\theta_2))\right)\end{array}\right)\\\cos\left(\begin{array}{l}(\omega-\omega_1)t\\+\left(\frac{\omega}{c}(x_2(\cos(\theta))-y_2\sin(\theta))-\frac{\omega_1}{c}(x_2\cos(\theta_1)+y_2\sin(\theta_1))\right)\end{array}\right)\end{array}\right)dt$$

$$=\frac{1}{32}+\frac{1}{32}$$

$$+\frac{1}{T_2}\int_0^{T_2}\left(\begin{array}{l}+\frac{1}{16}\cos\left(\begin{array}{l}(\omega-\omega_2)t-(\omega-\omega_1)t\\-\left(\frac{\omega}{c}(x_2(\cos(\theta))-y_2\sin(\theta))-\frac{\omega_2}{c}(x_2\cos(\theta_2)+y_2\sin(\theta_2))\right)\\-\left(\frac{\omega}{c}(x_2(\cos(\theta))-y_2\sin(\theta))-\frac{\omega_1}{c}(x_2\cos(\theta_1)+y_2\sin(\theta_1))\right)\end{array}\right)\\+\frac{1}{16}\cos\left(\begin{array}{l}(\omega-\omega_2)t+(\omega-\omega_1)t\\-\left(\frac{\omega}{c}(x_2(\cos(\theta))-y_2\sin(\theta))-\frac{\omega_2}{c}(x_2\cos(\theta_2)+y_2\sin(\theta_2))\right)\\+\left(\frac{\omega}{c}(x_2(\cos(\theta))-y_2\sin(\theta))-\frac{\omega_1}{c}(x_2\cos(\theta_1)+y_2\sin(\theta_1))\right)\end{array}\right)\end{array}\right)dt$$

$$= \frac{1}{16}$$

$$+\frac{1}{16}\cos\left(\begin{array}{l}(2\omega-\omega_2-\omega_1)t\\ -\left(-\frac{\omega_2}{c}\left(x_2\cos(\theta_2)+y_2\sin(\theta_2)\right)\right)\\ +\left(-\frac{\omega_1}{c}\left(x_2\cos(\theta_1)+y_2\sin(\theta_1)\right)\right)\end{array}\right)$$

$$=\frac{1}{8}\cos^2\left(\begin{array}{l}\left(\frac{2\omega-\omega_2-\omega_1}{2}\right)t\\ -\frac{1}{2}\left(-\frac{\omega_2}{c}\left(x_2\cos(\theta_2)+y_2\sin(\theta_2)\right)\right)\\ +\frac{1}{2}\left(-\frac{\omega_1}{c}\left(x_2\cos(\theta_1)+y_2\sin(\theta_1)\right)\right)\end{array}\right)$$

$$=\frac{1}{8}\cos^2\left(\begin{array}{l}\left(\frac{2\omega-\omega\gamma\left(1+\frac{v}{c}\cos(\theta)\right)-\omega\gamma\left(1-\frac{v}{c}\cos(\theta)\right)}{2}\right)t\\ -\frac{1}{2}\left(-\frac{\omega\gamma\left(1+\frac{v}{c}\cos(\theta)\right)}{c}\left(x_2\left(\frac{\cos\theta+\frac{v}{c}}{1+\frac{v}{c}\cos\theta}\right)+y_2\left(\frac{-\frac{1}{\gamma}\sin\theta}{1+\frac{v}{c}\cos\theta}\right)\right)\right)\\ +\frac{1}{2}\left(-\frac{\omega\gamma\left(1-\frac{v}{c}\cos(\theta)\right)}{c}\left(x_2\left(\frac{\cos\theta-\frac{v}{c}}{1-\frac{v}{c}\cos\theta}\right)+y_2\left(\frac{-\frac{1}{\gamma}\sin\theta}{1-\frac{v}{c}\cos\theta}\right)\right)\right)\end{array}\right)$$

$$=\frac{1}{8}\cos^2\left(\begin{array}{l}(1-\gamma)\omega t\\ -\frac{1}{2}\left(-\frac{\omega\gamma}{c}\left(x_2\left(\cos\theta+\frac{v}{c}\right)+y_2\left(-\frac{1}{\gamma}\sin\theta\right)\right)\right)\\ +\frac{1}{2}\left(-\frac{\omega\gamma}{c}\left(x_2\left(\cos\theta-\frac{v}{c}\right)+y_2\left(-\frac{1}{\gamma}\sin\theta\right)\right)\right)\end{array}\right)$$

$$=\frac{1}{8}\cos^2\left(\begin{array}{l}(\gamma-1)\omega t\\ +\frac{1}{2}\left(-\frac{\omega\gamma}{c}\left(x_2\left(\cos\theta+\frac{v}{c}\right)+y_2\left(-\frac{1}{\gamma}\sin\theta\right)\right)\right)\\ -\frac{1}{2}\left(-\frac{\omega\gamma}{c}\left(x_2\left(\cos\theta-\frac{v}{c}\right)+y_2\left(-\frac{1}{\gamma}\sin\theta\right)\right)\right)\end{array}\right)$$

$$=\frac{1}{8}\cos^2\left((\gamma-1)\omega t-\left(\frac{\omega\gamma}{c}\left(x_2\left(\frac{v}{c}\right)\right)\right)\right)$$

$$=\frac{1}{8}\cos^2\left((\gamma-1)\omega t-\frac{\omega\gamma v}{c^2}x_2\right)$$

したがって $\Psi_s=\pm\frac{1}{\sqrt{8}}\cos\left((1-\gamma)\omega t-\frac{\omega\gamma v}{c^2}x_2\right)$

となる。

Ψ_s を時間で微分すると

$$\frac{\partial\Psi_s}{\partial t}=-i(\gamma-1)\omega\Psi_s=-i\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}-1\right)\omega\Psi_s\cong-i\left(\left(1+\frac{v^2}{2c^2}\right)-1\right)\omega\Psi_s$$

$$=-i\frac{v^2}{2c^2}\omega\Psi_s$$

h はプランク定数　 f は振動数　 m は質量とし

$$\omega = 2\pi f = 2\pi \frac{mc^2}{h} = \frac{mc^2}{\hbar}$$

とすると

$$\frac{\partial \Psi_s}{\partial t} = -i \frac{v^2}{2c^2} \omega \Psi_s = -i \frac{v^2}{2c^2} \frac{mc^2}{\hbar} \Psi_s = -i \frac{1}{2} \frac{mv^2}{\hbar} \Psi_s$$

$E = \frac{1}{2} mv^2$ とすると

$$\frac{\partial \Psi_s}{\partial t} = -i \frac{E}{\hbar} \Psi_s$$

したがって

$$E\Psi_s = i\hbar \frac{\partial \Psi_s}{\partial t}$$

Ψ_s を距離で微分すると

$$\frac{\partial \Psi_s}{\partial x_2} = i \frac{\omega \gamma v}{c^2} \Psi_s \cong i \frac{\omega v}{c^2} \Psi_s = i \frac{mc^2}{\hbar} \frac{v}{c^2} \Psi_s = i \frac{mv}{\hbar} \Psi_s$$

p を運動量とし

$p = mv$ とすると

$$\frac{\partial \Psi_s}{\partial x_2} = i \frac{\omega \gamma v}{c^2} \Psi_s \cong i \frac{\omega v}{c^2} \Psi_s = i \frac{mc^2}{\hbar} \frac{v}{c^2} \Psi_s = i \frac{mv}{\hbar} \Psi_s = i \frac{p}{\hbar} \Psi_s$$

したがって

$$-i\hbar \frac{\partial \Psi_s}{\partial x_2} = p\Psi_s$$

以上より

$$i\hbar \frac{\partial \Psi_s}{\partial t} = \frac{1}{2} mv^2 \Psi_s$$

$$\frac{\partial^2 \Psi_s}{\partial {x_2}^2} = -\frac{m^2 v^2}{\hbar^2} \Psi_s$$

となるので

$$i\hbar \frac{\partial \Psi_s}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi_s}{\partial {x_2}^2}$$

となってシュレーディンガー方程式と一致する。