A derivation of Klein–Gordon equation from the standing wave (1dimension)

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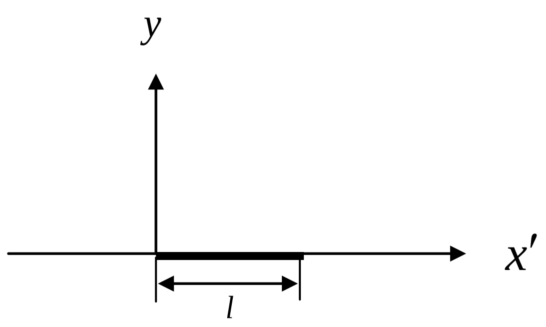
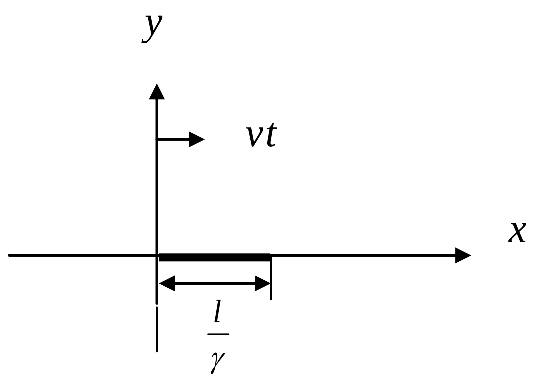
I assume that there is a standing wave solution of non-linear wave equation (see[1])in a reference frame that moves toward x .

If we look at the standing wave from the moving reference frame it seems as following.



When l is a length of one wave length.

If we look at the standing wave from the static reference frame it seems as following.



From the Lorentz transformation.

$$x' = \gamma(x - vt)$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

Where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

If E is a standing wave

$$E = f(x', y\ )\cos(\omega t')\cos\left(\frac{\omega x'}{c}\right)$$

However here is approximately $f(x', y) = f_0$ Where f_0 is a constant.

$$E = f_0 \cos(\omega t')\cos\left(\frac{\omega x'}{c}\right)$$

If Ψ_G is a Fourier's coefficient of $\cos\left(\frac{\omega x'}{c}\right)$ about the standing wave E 。

$$\Psi_G = \frac{\omega}{\pi}\int_{x'=-\frac{\pi}{\omega}}^{\frac{\pi}{\omega}} E\cos\left(\frac{\omega x'}{c}\right)dx' = f_0\cos(\omega t')$$

$$= f_0\cos\left(\omega\gamma\left(t - \frac{vx}{c^2}\right)\right)$$

$$= f_0\cos\left(\omega\gamma t - \frac{\omega\gamma x}{c}\left(\frac{v}{c}\right)\right)$$

Therefore

$$-\frac{1}{c^2}\frac{\partial^2\Psi_G}{\partial t^2}+\nabla^2\Psi_G=\left(\frac{1}{c^2}\omega^2\gamma^2-\frac{\omega^2\gamma^2}{c^2}\frac{v^2}{c^2}\right)\Psi_G$$

$$=\frac{\omega^2}{c^2}\gamma^2\left(1-\frac{v^2}{c^2}\right)\Psi_G=\frac{\omega^2}{c^2}\left(\frac{1-\frac{v^2}{c^2}}{1-\frac{v^2}{c^2}}\right)\Psi_G=\frac{\omega^2}{c^2}\Psi_G$$

If h is the plank constant. f is the frequency, m is the mass,

If $\omega=2\pi f=2\pi\frac{mc^2}{h}=\frac{mc^2}{\hbar}$ then

$$-\frac{1}{c^2}\frac{\partial^2\Psi_G}{\partial t^2}+\nabla^2\Psi_G=\frac{1}{c^2}\frac{m^2c^4}{\hbar^2}\Psi_G=\frac{m^2c^2}{\hbar^2}\Psi_G$$

Therefore

$$-\frac{1}{c^2}\frac{\partial^2\Psi_G}{\partial t^2}+\nabla^2\Psi_G=\frac{m^2c^2}{\hbar^2}\Psi_G$$

As a result, it consistent to Klein–Gordon equation.

Reference

1. Tetsuya Nagai "A standing wave solution of non-linear wave equation[A condition when the frequency is in proportion to the weight]" http://www.tegakinet.jp/wave/stand.pdf