(3.19)式からの(8.15)式の導出

$\varphi(x) = \int \widetilde{dk}\left[a(\mathbf{k})e^{ikx} + a^*(\mathbf{k})e^{-ikx}\right]$　(3.19)

$a^*(\mathbf{k}) \equiv a^\dagger(\mathbf{k})$として

$\varphi(x) = \int \widetilde{dk}\left[a(\mathbf{k})e^{ikx} + a^\dagger(\mathbf{k})e^{-ikx}\right]$

$a(\mathbf{k})$は消滅演算子

$a^\dagger(\mathbf{k})$は生成演算子なので

$\left\langle 0\middle|a^\dagger(\mathbf{k}) = a(\mathbf{k})\middle|0\right\rangle = 0$

ボゾンとして

$[a(\mathbf{k}), a^\dagger(\mathbf{k}')] = (2\pi)^3 2\omega\delta(\mathbf{k}-\mathbf{k}')$　(3.29) なので

$$\left\langle 0\middle|a(\mathbf{k})a^\dagger(\mathbf{k})\middle|0\right\rangle = \left\langle 0\middle|a^\dagger(\mathbf{k})a(\mathbf{k})\middle|0\right\rangle - \left\langle 0\middle|a^\dagger(\mathbf{k})a(\mathbf{k})\middle|0\right\rangle = \left\langle 0\middle|[a^\dagger(\mathbf{k}), a(\mathbf{k})]\middle|0\right\rangle$$
$$= (2\pi)^3 2\omega\delta(\mathbf{k}-\mathbf{k}')$$

$t_1 > t_2$ のとき

$$\langle 0|T\varphi(x_1)\varphi(x_2)|0\rangle = \langle 0|\varphi(x_1)\varphi(x_2)|0\rangle$$
$$= \left\langle 0\middle|\left(\int \widetilde{dk}\left[a(\mathbf{k})e^{ikx_1} + a^\dagger(\mathbf{k})e^{-ikx_1}\right]\right)\left(\int \widetilde{dk'}\left[a(\mathbf{k}')e^{ik'x_2} + a^\dagger(\mathbf{k}')e^{-ik'x_2}\right]\right)\middle|0\right\rangle$$

$$= \left\langle 0\middle|\left(\int \widetilde{dk}\left[a(\mathbf{k})e^{ikx_1}\right]\right)\left(\int \widetilde{dk'}\left[a(\mathbf{k}')e^{ik'x_2}\right]\right)\middle|0\right\rangle$$
$$+ \left\langle 0\middle|\left(\int \widetilde{dk}\left[a(\mathbf{k})e^{ikx_1}\right]\right)\left(\int \widetilde{dk'}\left[a^\dagger(\mathbf{k}')e^{-ik'x_2}\right]\right)\middle|0\right\rangle$$
$$+ \left\langle 0\middle|\left(\int \widetilde{dk}\left[a^\dagger(\mathbf{k})e^{-ikx_1}\right]\right)\left(\int \widetilde{dk'}\left[a(\mathbf{k}')e^{ik'x_2}\right]\right)\middle|0\right\rangle$$
$$+ \left\langle 0\middle|\left(\int \widetilde{dk}\left[a^\dagger(\mathbf{k})e^{-ikx_1}\right]\right)\left(\int \widetilde{dk'}\left[a^\dagger(\mathbf{k}')e^{-ik'x_2}\right]\right)\middle|0\right\rangle$$
$$= \left\langle 0\middle|\left(\int \widetilde{dk}\left[a(\mathbf{k})e^{ikx_1}\right]\right)\left(\int \widetilde{dk'}\left[a^\dagger(\mathbf{k}')e^{-ik'x_2}\right]\right)\middle|0\right\rangle$$
$$= \left\langle 0\middle|\int \widetilde{dk}\int \widetilde{dk'}\left[a(\mathbf{k})e^{ikx_1}a^\dagger(\mathbf{k}')e^{-ik'x_2}\right]\middle|0\right\rangle$$
$$= \left\langle 0\middle|\int \widetilde{dk}\int \widetilde{dk'}\left[a(\mathbf{k})a^\dagger(\mathbf{k}')\right]e^{i(kx_1-k'x_2)}\middle|0\right\rangle$$

$$= \int \widetilde{dk} \int \widetilde{dk'}\ (2\pi)^3 2\omega\delta(\mathbf{k}-\mathbf{k'}\ )e^{i(kx_1 - k'\ x_2)}$$

$$= \int \widetilde{dk} \int \frac{d^3k'}{(2\pi)^3 2\omega}(2\pi)^3 2\omega\delta(\mathbf{k}-\mathbf{k'}\ )e^{ik(x_1 - k'\ x_2)}$$

$$= \int \widetilde{dk}\, e^{ik(x_1 - x_2)}$$

ただし

$\widetilde{dk} \equiv \frac{d^3k}{(2\pi)^3 2\omega}$　(3.18)を使用している。

同様に

$t_1 < t_2$ のとき

$$\langle 0|T\varphi(x_1)\varphi(x_2)|0\rangle = \langle 0|\varphi(x_2)\varphi(x_1)|0\rangle = \int \widetilde{dk}\, e^{ik(x_2 - x_1)}$$

従って

$$\langle 0|T\varphi(x_1)\varphi(x_2)|0\rangle = \theta(t_1 - t_2)\int \widetilde{dk}\, e^{ik(x_1 - x_2)} + \theta(t_2 - t_1)\int \widetilde{dk}\, e^{ik(x_2 - x_1)}$$

$$= \frac{1}{i}\Delta(x_2 - x_1) \quad (8.15)$$

ただし$x_2 \equiv x$, $x_1 \equiv x'$ とし

$\Delta(x - x'\ ) = i\theta(t - t'\ )\int \widetilde{dk}\, e^{ik(x - x'\ )} + i\theta\left(t' - t\right)\int \widetilde{dk}\, e^{-ik(x - x'\ )}$　(8.13)